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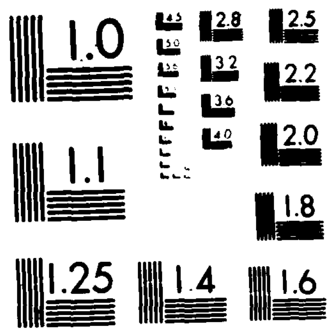

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Final Technical Report
March 1987

EFFICIENT COMPLEX MATRIX MULTIPLICATION

University of Buffalo

Adly T. Fam

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ROME AIR DEVELOPMENT CENTER
Air Force Systems Command
Griffiss Air Force Base, NY 13441-5700

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Complex Matrix Multiplication
Efficient Multiplication Algorithm

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

A well known algorithm for complex multiplication which requires three real multiplications and five real additions is observed not to require commutativity. This extends its applicability to complex matrices as examined in this report. The computational savings are shown to approach 1/4, even if a real multiplication is not more computationally costly than a real addition. The computational cost function used is based on the number of equivalent real additions, with every real multiplication counted as equivalent to r real additions.

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I. INTRODUCTION

Multiplication of the complex numbers x and y, where $x = a+jb$ and $y = c+jd$ requires the computation of $ac-bd$ and $ad+bc$. If computed directly, this requires four real multiplications and two real additions. It is well known, as frequently attributed to Golub, that the identity

$$xy = (ac-bd)+j(ad+bc)$$
$$= (a(c-d)+(a-b)d)+j(b(c+d)+(a-b)d) \qquad (1)$$

could be used, requiring three real multiplications and five real additions instead. This identity could result as a special case of application of an efficient algorithm of polynomial multiplication as discussed elegantly by Winograd in [1], page 18. Let a real multiplication be computationally equivalent to r real additions. Clearly, application of (1) is of interest only if $r>3$, as indicated by Moharir in [2]. With the advent of distributed computing, and the increased computational power available on individual VLSI chips, the value of r approaches unity in some cases. This is the case, for example, in applications where the predominant factor in the computational cost is that of the I/O requirements and data manipulation.

An important field in which multiplication is inherently more costly than addition is that of matrix arithmetic. For nxn real matrices, a multiplication requires $O(n^3)$ operations, while only $O(n^2)$ are needed for addition. Fortunately, commutativity is not required for (1) to hold, and (1) is therefore applicable to complex matrices with compatible dimensions. In Section II, the case of square complex matrices is considered where application

of (1) is shown to result in saving up to 1/4 of the computations, even if $r = 1$.

The three additions in (1) depend on either x or y, but not both. The quantity (a-b) depends only on x, while (c+d) and (c-d) require only y. Such computations have the desirable feature that they do not require data communication to combine x and y. In addition, if either x or y is fixed, such quantities could be precomputed only once. There is an asymmetry in above quantities since only one of them depends on x, while the other two depend on y. This asymmetry suggests the existence of a dual form, where the roles of x and y are interchanged, but without requiring commutativity. This form is

$$xy = ((a-b)c+b(c-d))+j((a+b)d+b(c-d)) \qquad (2)$$

which is of importance for rectangular matrices and applications with fixed data as discussed in Section III. The Conclusion in Section IV, Comments on some applications and on the possibility of combining this work with other matrix multiplication algorithms are presented.

II. SQUARE MATRICES

In this section, the x and y of (1) and (2) represent nxn complex matrices. Direct multiplication of x and y requires A real additions and M real multiplications, where

$$A = 2n^2+4n^2(n-1) = 4n^3-2n^2,$$
$$M = 4n^3 \qquad (3)$$

On the other hand using (1) or (2) requires

$$A = 5n^2+3n^2(n-1) = 3n^3+2n^2,$$
$$M = 3n^3 \qquad (4)$$

The cost of computing xy in equivalent additions is

$$C = 4n^3(1+r)-2n^2 \tag{5}$$

for direct computation, and

$$C' = 3n^3(1+r)+2n^2 \tag{6}$$

for computation using (1) or (2). To compare the two approaches we use either

$$S = (C-C')/C = (n(1+r)-4)/(4n(1+r)-2) \tag{7}$$

which represents the relative reduction in computational cost when (1) or (2) is used for complex square matrix multiplication, or

$$R = 1-S = C'/C = (3n(1+r)+2)/(4n(1+r)-2) \tag{8}$$

the ratio of their costs. It is clear that as n(1+r) increases, S approaches 1/4, while R approaches 3/4. The following table should be of value in assessing the range of values of r and n for which the approach is of interest. The values of R in the table are rounded to two decimal locations. Even if multiplication is considered computationally equivalent to addition, i.e. for r = 1, appreciable savings are possible for modest values of n. For n = 2 direct computation is equivalent to the proposed approach, for which R = .91 and decreases further with increasing n to approach 3/4.

r\n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1.3	1	.91	.87	.84	.83	.81	.81	.80	.79	.79	.79	.78	.78	.78
2	1.1	.91	.85	.83	.81	.80	.79	.79	.78	.78	.78	.77	.77	.77	.77
3	1	.87	.83	.81	.79	.79	.78	.78	.77	.77	.77	.77	.77	.77	.76
10	.83	.79	.78	.77	.77	.76	.76	.76	.76	.76	.76	.76	.76	.76	.76

Table for R as a Function of n and r.

III. THE GENERAL CASE

For the general case of complex matrices x and y of dimensions pxn and nxm, respectively, direct computation requires

$$A = 2pm+4pm(n-1) = 4pmn-2pm,$$

$$M = 4pmn \qquad (9)$$

For non-square matrices (1) and (2) yield different results. If (1) is used we get

$$A = 2pn+mn+2pm+3pm(n-1) = 3pmn+2pn+mn-pm,$$

$$M = 3pmn \qquad (10)$$

while (2) results in

$$A = pn+2mn+2pm+3pm(n-1) = 3pmn+pn+2mn-pm \qquad (11)$$

and the same value of M = 3pmn. One pn in the expression of A in (9) is replaced by an mn in (11). Therefor, (1) should be used for matrix pairs with $p<m$ and (2) for those with $m<p$. Direct computation requires

$$C = 4pmn(1+r)-2pm \qquad (12)$$

equivalent additions, while the proper choice of (1) or (2) results in

$$C' = 3pmn(1+r)-pm+mn+pn+n(\min\{p,m\}) \qquad (13)$$

equivalent additions. From (12) and (13) we obtain the ratio

$$R = C'/C$$

$$= (3pmn(1+r)-pm+mn+pn+n(\min\{p,m\}))/(4pmn(1+r)-2pm) \qquad (14)$$

which approaches 3/4 as pmn(1+r) increases.

In some cases, one of the two matrices either remains constant or changes infrequently, while the second matrix changes frequently. Let x be fixed, and y frequently changing. For direct computation, C is the same as in (13) since every

multiplication or addition involves at least one element of y. Computation based on (2) requires the calculation of a+b and a-b which involves only the elements of x and could be precomputed and are therefore not included in assessing the computational cost next. All other computations involve y and require

$$A = mn+2pm+3pm(n-1),$$

$$M = 3pmn \tag{15}$$

resulting in

$$C' = 3pmn(1+r)+mn-pm \tag{16}$$

and

$$R = (3n(1+r)+n/p-1)/(4n(1+r)-2) \tag{17}$$

which does not depend on m. Even for $r = 1$, R in (17) is always ≤ 1 with the limit $R = 1$ attained for $n = p = 1$, in which case a complex multiplication costs three real additions and three real multiplications.

The case of an inner product is of particular interest. For $p = m = 1$, $r = 1$, and large n we get $R = 7/8$ from (17). This is in comparison to the case where both x and y are not fixed, resulting in $R = 1$ from (14).

IV. CONCLUSION

Extension of (1) and (2) to complex matrices resulting in computational savings of up to 1/4 could be of interest in a variety of applications. In digital signal processing, inner products, vector-scalar and vector-matrix multiplications are some times encountered with complex entries. This is the case, for example in polyphase filters and filter banks and some signal transforms. This is also the case in radar and communication applications, with digital processing in the base-band.

Efficient algorithms for real matrix multiplication could be advantageously combined with this work. For example, the coefficient of the $O(n^{\log_2 7})$ algorithm of Strassen in [3] would be reduced by up to 1/4.

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